

Geometrization of the Quantum Effects

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February 1, 2008

Abstract

We present a conformally invariant generalized form of the free particle action by connecting the wave and particle aspects through gravity. Conformal invariance breaking is introduced by choosing a particular configuration of dynamical variables. This leads to the geometrization of the quantum aspects of matter.

1 Introduction

The general point of view of wave-particle duality proposed by de Broglie ¹ considers all atomic objects such as photons, electrons, protons, etc. as consisting of the physical association of two entities: (a) a wave, devoid of energy and momentum but nevertheless objectively real and propagating in space-time; (b) a particle, as a single tiny region of highly localized energy incorporated in the wave, like a singularity in motion ². In fact corpuscle serves as carrier of the energy belonging to the wave, while the wave constitutes an extended entity surrounding the particle. Generally, wave and particle are considered as different manifestations of a single system. In particular, it is claimed they are two phenomena of essentially different nature which constitutes the complementary aspects of a single system.

The implementation of wave-particle duality in the causal interpretation of quantum mechanic ³ leads to a distinct feature, because one finds a direct interplay between particle properties and those of waves through the quantum potential. This would imply a substantial connection between wave and particle in a typical dynamical process in quantum mechanics. In particular one can infer the existence of the wave from the observation on the associated particle. This paper undertakes a preliminary step towards connecting this distinct face of wave-particle duality with conformal invariance. We shall study the conformal invariant coupling of a relativistic particle action to a scalar field through the gravity. For the resulting theory, we identify two different conformal frames. In the first frame, which we call classical, the usual particle properties are established. We shall then define a second frame, which we call quantum, in

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terms of a local change of the classical frame. In this frame we establish a dynamical interplay between a particle and the applied conformal factor. This suggests that the association of a wave with a particle in the causal interpretation of quantum mechanics may be an irreducible effect of a conformal invariant coupling of the particle to gravity. We shall use units in which $\hbar = c = 1$.

2 The Conformal invariant coupling of a Particle to Gravity

We consider a free particle action functional consisting of a real scalar field ϕ , as follows

$$S_p[\phi] = -\frac{1}{2} \int d^4x \sqrt{-g} (\phi^2 g_{\mu\nu} \partial^\mu S \partial^\nu S + \frac{\lambda}{2} \phi^4) \quad (1)$$

where $g_{\mu\nu}$ is the Riemannian metric, S is a Hamilton-Jacobi function associated with the motion of free particle and λ is a dimensionless parameter which depends on the particle properties. It must be noted that, the action (1) dose not have the kinetic energy of the scalar field ϕ . In order to complete the action (1), one may consider the kinetic term as an action functional which contains the real scalar field ϕ and the gravitational field in the form

$$S_w[\phi] = -\frac{1}{2} \int d^4x \sqrt{-g} (g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi + \frac{1}{6} R \phi^2). \quad (2)$$

Here R is the Ricci curvature associated with the Riemannian metric ‡ . Thus the total action is obtained by taking (1) and (2) together, giving us

$$\begin{aligned} S[\phi] &= S_p[\phi] + S_w[\phi] \\ &= -\frac{1}{2} \int d^4x \sqrt{-g} \{ g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi + (g_{\mu\nu} \partial^\mu S \partial^\nu S + \frac{1}{6} R) \phi^2 + \frac{\lambda}{2} \phi^4 \}. \end{aligned} \quad (3)$$

Variations of $S[\phi]$ with respect to ϕ gives

$$\square^g \phi - (g_{\mu\nu} \partial^\mu S \partial^\nu S + \frac{1}{6} R) \phi - \lambda \phi^3 = 0$$

or equivalently

$$g_{\mu\nu} \partial^\mu S \partial^\nu S = -M^2 + \frac{\square^g \phi}{\phi} - \frac{1}{6} R \quad (4)$$

where $M = (\lambda \phi^2)^{1/2}$ represents a varying mass-parameter, and \square^g denotes the d'Alembertian associated with the Riemannian metric $g_{\mu\nu}$, namely

$$\square^g = (-g)^{-\frac{1}{2}} \partial_\mu [(-g)^{\frac{1}{2}} g^{\mu\nu} \partial_\nu].$$

We call Eq. (4), the generalized Hamilton-Jacobi equation for a particle, in which the particle's S-function is dynamically coupled to gravity and the scalar field ϕ .

Variations of $S[\phi]$ with respect to $g_{\mu\nu}$ and S leads to

$$\begin{aligned} \phi^2 G_{\mu\nu} + (g_{\mu\nu} \square^g - D_\mu D_\nu) \phi^2 + 6(D_\mu \phi D_\nu \phi + \phi^2 D_\mu S D_\nu S) \\ - 3g_{\mu\nu} (D_\alpha \phi D^\alpha \phi + \phi^2 D_\alpha S D^\alpha S) - \frac{3}{2} \lambda \phi^4 g_{\mu\nu} = 0 \end{aligned} \quad (5)$$

[‡]Our sign convention is given by $R_{\mu\nu} \sim \Gamma_{\mu\nu,\alpha}^\alpha - \Gamma_{\mu\alpha,\nu}^\alpha$ and the signature is $(-, +, +, +)$.

and

$$\partial_\mu(\sqrt{-g}\phi^2 g^{\mu\nu}\partial_\nu S) = 0 \quad (6)$$

where in Eq. (5), $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ is the Einstein tensor, and D_μ represents the covariant derivative. One may notice that the dynamical coupling of the particle's S-function to gravity is entirely encoded in the Eqs. (5) and (6) only. In fact the generalized Hamilton-Jacobi equation (4) contains no new informations. Actually, we may take the trace of Eq. (5) to obtain

$$\phi\{\square^g\phi - (g_{\mu\nu}\partial^\mu S\partial^\nu S + \frac{1}{6}R)\phi - \lambda\phi^3\} = 0$$

which, leads directly to the Eq. (4). This feature is a consequence of the fact that the total action (3) is exactly invariant under the conformal transformation

$$\begin{cases} g_{\mu\nu} \longrightarrow \Omega^2(x)g_{\mu\nu} \\ \phi \longrightarrow \Omega^{-1}(x)\phi \end{cases} \quad (7)$$

This implies that the theory introduced by the action (3) can be considered in various (conformal) frames depending on the particular choice of the local standard of length ^{4,5}. A simple conformal frame may be characterized by the condition $\phi = \phi_0 = \text{const.}$ We may call it the classical frame. In this frame Eqs. (5) and (6) take, respectively, the forms

$$G_{\mu\nu} + 6D_\mu S D_\nu S - 3g_{\mu\nu}D_\alpha S D^\alpha S - \frac{3}{2}\lambda\phi_0^2 g_{\mu\nu} = 0 \quad (8)$$

and

$$\square^g S = 0. \quad (9)$$

These equations contain the entire dynamical coupling of the particle's S-function to gravity in the classical frame. In this frame we find, by taking the trace of Eq. (8), the generalized Hamilton-Jacobi equation

$$g_{\mu\nu}\partial^\mu S\partial^\nu S = -M_0^2 - \frac{1}{6}R \quad (10)$$

with $M_0 = (\lambda\phi_0^2)^{1/2}$ being a constant mass-parameter.

A varying configuration of the scalar field ϕ relative to the constant value $\phi = \phi_0$ can be chosen as a representation of a new frame which we may call the quantum frame. Such a varying configuration can represent fluctuations of the scalar field ϕ around the constant value ϕ_0 . These fluctuations may be described in terms of a local change of the classical frame. To be specific, one may define a quantum frame by applying a conformal transformation (7) to the classical frame with

$$\Omega(x) = \frac{\phi_0}{\phi}.$$

Using the transformation law of the Ricci tensor $R_{\mu\nu}$ under (7), we then find ⁶

$$R_{\mu\nu} \longrightarrow R_{\mu\nu} + \Omega^{-2}\{4D_\mu\Omega D_\nu\Omega - 2\Omega D_\mu D_\nu\Omega - g_{\mu\nu}g^{\alpha\beta}(\Omega D_\alpha D_\beta\Omega + D_\alpha\Omega D_\beta\Omega)\}$$

$$G_{\mu\nu} \longrightarrow G_{\mu\nu} + \Omega^{-2}\{(g_{\mu\nu}\square^g - D_\mu D_\nu)\Omega^2 + 6D_\mu\Omega D_\nu\Omega - 3g_{\mu\nu}D_\alpha\Omega D^\alpha\Omega\}.$$

Consequently the Eq. (8) transforms as

$$\begin{aligned} \Omega^2 G_{\mu\nu} + (g_{\mu\nu} \square^g - D_\mu D_\nu) \Omega^2 + 6(D_\mu \Omega D_\nu \Omega + \Omega^2 D_\mu S D_\nu S) \\ - 3g_{\mu\nu} (D_\alpha \Omega D^\alpha \Omega + \Omega^2 D_\alpha S D^\alpha S) - \frac{3}{2} M_0^2 \Omega^2 g_{\mu\nu} = 0. \end{aligned} \quad (11)$$

Taking the trace of Eq. (11), we find

$$g_{\mu\nu} \partial^\mu S \partial^\nu S = -M_0^2 + \frac{\square^g \Omega}{\Omega} - \frac{1}{6} R. \quad (12)$$

Comparing Eqs. (10) and (12), we find that, in the quantum frame the Hamilton-Jacobi equation is identical with the classical frame, except for the particle mass which is modified by an extra term $\frac{\square^g \Omega}{\Omega}$. The appearance of this term is a consequence of the local change of the classical frame by the conformal transformation (7). The interpretation of this extra term is postponed to the next section.

It must be pointed out however, that Eq. (9) is left unchanged in the quantum frame. It is convenient to rewrite it in the following form for later usage

$$\partial_\mu (\sqrt{-g} \Omega^2 g^{\mu\nu} \partial_\nu S) = 2\sqrt{-g} \mathcal{M} \Omega^2 \frac{d}{d\tau} \ln \Omega \quad (13)$$

where we have used

$$\partial_\mu S = \mathcal{M} U_\mu \quad \text{and} \quad U_\mu \partial^\mu = \frac{d}{d\tau}$$

in which τ is a parameter along the particle trajectory and the value \mathcal{M} is dependent on the frame one is using.

3 Derivation of a pilot wave

In the quantum frame, it is possible to assign a pilot wave to the particle motion described by the Hamilton-Jacobi equation (12). Consider the wave

$$\psi = \Omega e^{iS}$$

which satisfies, as a consequence of Eqs. (12) and (13), the equation

$$\square^g \psi - M_0^2 \psi = \left(\frac{1}{6} R + 2i\mathcal{M} \frac{d}{d\tau} \ln |\psi| \right) \psi.$$

In the background approximation $g_{\mu\nu} \longrightarrow \eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski metric, and in the limit in which any conceivable dependence of the conformal factor Ω on the parameter τ along the particle trajectory is ignored, this equation reduces to the massive Klein-Gordon equation

$$\square \psi - M_0^2 \psi = 0$$

where, \square denotes the d'Alembertian associated with the Minkowski-metric. The merit of introducing the wave ψ is that it acts as a sort of a pilot wave in the sense of causal interpretation of quantum mechanics^{3,7}, the term $\frac{\square^g \Omega}{\Omega}$ on the right hand side of Eq. (12) being the associated quantum potential.

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